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DEPARTMENT OF ECONOMICS

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PREFERENCES IN POLICY OPTIMIZATION
AND OPTIMAL ECONOMIC POLICY

by

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The undulating Tuscan hills and full-blooded Chianti Classico's - and other 'classics' - were instrumental in initiating the work reported here. However the more solid encouragement provided by Professors Richard Goodwin and Jean Paul Fitoussi was almost equally relevant. Jessica Spataro deciphered efficiently, as always, the hireoglyphics. Alas, none of the worthies mentioned above are responsible for the remaining infelicities.

BADIA FIESOLANA, SAN DOMENICO DI FIESOLE



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§ 1. Introduction

In a stimulating article in the KYKLOS, John H. Makin discussed the important question of obtaining information about 'the relative or absolute weights attached by policy makers to stated target variables' (Makin (1976) p. 709). His approach to the problem, following an empirical tradition richly referenced in his article, was to investigate the identifiability conditions of a 'reaction' function parametrized by the weights of a preference relation and linking, as variables, targets and instruments. Next, under the assumed identifiability conditions, standard econometric techniques were to be applied to derive estimates of the parameters.

He is, however, aware and candid enough to admit that serious difficulties beset the above approach due to 'the implicit assumptions made that the policy-maker has a correct view of the impact of policy instruments upon targets' (Makin, op. cit., p. 724). Indeed, as he points out, some authors even go as far as rejecting the whole methodology of optimal policy derivation in view of the above implicit assumption (quite independently of the separate set of criticism levelled by the New Classical Macroeconomists!) As Makin observes - and here it is perhaps worthwhile quoting him in full - the difficulties implied by the above implicit assumption :

".... really points to the more fundamental problem of just how it is that the policy maker comes to obtain exogenous information on the reduced form coefficients in / the reaction function/. Direct estimation is ruled out because the data has already been employed, with instruments in the role of endogenous variables and targets in the role of exogenous variables, to estimate the reaction function. Tinbergen ...

termed the problem ... the 'conformity' concept and noted that trial and error may be necessary for policy makers to obtain perceived values of reduced form coefficients in the [Coefficient matrix of the reaction function] that are close to the true values".

(Makin, op. cit., p. 725, italics added)

In fact Ragnar Frisch in two of his very last publications was even more emphatic on the necessity for policy-makers to obtain such information (cf. Frisch (1972), 1981). His 'plea for a new type of cooperation between politicians and econometricians' so as to formalize 'the preference function which must underlie the very concept of an optimal economic policy' (Frisch, 1972, p. 5) seems to have been completely ignored. The method he envisaged was, to a large extent, similar to Tinbergen's advocacy of the 'trial and error' method - and indeed was an attempt to formalize it as a convergent iterative process.

In this paper we attempt to tackle the problems Makin has noted along the lines suggested by Tinbergen and Frisch by formalizing the cooperation between politicians and econometricians as an iterative - or trial and error - process such that the weights in the preference functions can be obtained in a finite number of steps.

§ 2. The Problem of Formalizing the Preference Function and an Iterative Solution to it.

a) General Remarks:

Given the description of an economy in the form of a standard econometric model, achievement of desired targets depend,

essentially, on the weighting of these targets and the constraints implied by the model. It is clear that only in the course of formulating and implementing policy decisions that policy makers obtain an increasingly concrete and accurate knowledge of what can be done. As a result of this, a realization of what is desired gains a sharper focus in the sense that the different targets would be reweighted in terms of each other ⁽¹⁾. If, therefore, the Frisch-Tinbergen suggestion is to be feasible, the real-time process of formulating and implementing policy decisions and then revising the relative importance to be attributed to targets in the light of realized values must be substituted by a conceptual, albeit implementable, iterative process replicating the above actions ⁽²⁾.

There is, of course, no such thing as a 'best' set of weights independent of a 'best' desired path. In the context of optimal economic policy what is arrived at is a politically acceptable path for target variables - but optimally generated. Since, as Makin also notes (cf. Makin, op. cit., p. 727), it would be unrealistic to expect politicians and policy makers to directly reveal their priorities (even if they could!), an indirect approach would be not only desirable but imperative. Frisch's suggestion is that the trial and error process of real-time cognition of the relationship between weights, instruments and targets should be substituted by the formalization of cooperation between policy makers and econometricians (model builders) such that the policy maker (unwittingly !) reveals his priorities. The cooperation itself is to take the form of the econometrician or model builder, for an assumed arbitrary (say, identical) set of weights, optimally solving for the target variables and confronting the policy maker with the relevant values.

Once these initial 'optimal values' have been determined the method we outline below provides a systematic way in which the policy maker can state his dissatisfaction with the values of the various target variables and tell the model builder / econometrician how he would like it changed. ("I do not like such a high current account deficit - please, also note that I cannot carry my cabinet colleagues on such levels for the government deficit"!!). This leaves the econometrician with the difficult - but, as we shall show, not impossible - task of reweighting the targets such that a more (politically) acceptable set of values can be obtained. It must be noted that almost any statement by the policy maker implies a relative weighting. The task is to formalize this implied set of weights. The amendments to the (optimal) values of the target variables (to make them more acceptable politically) are translated into corresponding corrections to the weighting matrix ⁽³⁾. In the method of formalized cooperation that is described below an 'optimal' set of weights may be obtained by repeating the above procedure so that, at the end of this iterative procedure, the 'final' set of values for the target variables will be acceptable to the policy maker.

b) Formalization of the Problem:

Consider the problem introduced by Makin

$$\min \{ J(\underline{Y}, \underline{U}) \mid (\underline{Y}, \underline{U}) \in R \} \quad (2.1)$$

where the objective function J is a quadratic given by

$$J(\underline{Y}, \underline{U}) \triangleq \frac{1}{2} \begin{bmatrix} \underline{Y} - \underline{Y}^d \\ \underline{U} - \underline{U}^d \end{bmatrix}^T Q \begin{bmatrix} \underline{Y} - \underline{Y}^d \\ \underline{U} - \underline{U}^d \end{bmatrix} \quad (2.2)$$

with Q a symmetric ⁽⁴⁾ positive definite ⁽⁵⁾ matrix and the feasible region R given by

$$R \triangleq \{ (\underline{Y}, \underline{U}) \mid N^T \begin{bmatrix} \underline{Y} \\ \underline{U} \end{bmatrix} = \underline{b} \} \quad (2.3)$$

(corresponding to, for example, equations (1) and (2) in Makin (op. cit.), but in a more generalized form) with the matrix N denoting the model Jacobian. Without loss of generality, we assume that the columns of N are linearly independent vectors ⁽⁶⁾. The vector \underline{Y} denotes the endogenous variable vector and \underline{U} denotes the vector of policy instruments ⁽⁷⁾.

The superscript d on \underline{Y} and \underline{U} in (2.2) denotes values of \underline{Y} and \underline{U} respectively which are desirable for the economic system. However, \underline{Y}^d and \underline{U}^d are not necessarily feasible. Indeed if $(\underline{Y}^d, \underline{U}^d) \in R$, then (2.1) has the obvious solution $\underline{Y}_* = \underline{Y}^d$ and $\underline{U}_* = \underline{U}^d$. It is when $\underline{Y}^d, \underline{U}^d \notin R$ that it becomes important for the policy maker to specify the importance he attaches in Y_i , the i th element of \underline{Y} , attaining its corresponding desired value Y_i^d versus Y_j attaining Y_j^d . Similarly, the importance in U_m attaining U_m^d versus U_n attaining U_n^d needs to be specified. Finally, the trade-off between Y_i attaining Y_i^d and U_m attaining U_m^d has to be specified. All this preference information is clearly summarized in the specification of the matrix Q in (2.2) which has a slightly generalized version of the structure adopted by Makin who has assumed that the trade-off between Y_i attaining Y_i^d and U_m attaining U_m^d is zero. This amounts to assuming that the corresponding off-diagonal submatrices of Q are zero.

The formulation (21.) is essentially a quadratic programming problem. This framework can also be suitably modified to

take account of linear terms in the quadratic objective function by a simple redefinition of desired values in (2.2). Consider the objective function

$$J_e(\underline{Y}, \underline{U}) \triangleq \underline{e}^T \begin{bmatrix} \underline{Y} - \underline{Y}_e^d \\ \underline{U} - \underline{U}_e^d \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \underline{Y} - \underline{Y}_e^d \\ \underline{U} - \underline{U}_e^d \end{bmatrix}^T Q \begin{bmatrix} \underline{Y} - \underline{Y}_e^d \\ \underline{U} - \underline{U}_e^d \end{bmatrix} \quad (2.4)$$

where Q is symmetric positive definite and \underline{e} is a constant vector. The unconstrained minimum of (2.4) is given by

$$\begin{bmatrix} \underline{Y}^u \\ \underline{U}^u \end{bmatrix} = \begin{bmatrix} \underline{Y}_e^d \\ \underline{U}_e^d \end{bmatrix} - H \underline{e} \quad (2.5)$$

with $H = Q^{-1}$. From an inspection of first order optimality conditions, it can be seen that \underline{Y}^* , \underline{U}^* which solve

$$\min \{ J_e(\underline{Y}, \underline{U}) \mid (\underline{Y}, \underline{U}) \in R \} \quad (2.6)$$

also solve (2.1) when the desired values in (2.2) are defined as

$$\begin{bmatrix} \underline{Y}^d \\ \underline{U}^d \end{bmatrix} = \begin{bmatrix} \underline{Y}^u \\ \underline{U}^u \end{bmatrix} \quad (2.7)$$

Thus, (2.6) can be straightforwardly expressed as (2.1) using (2.7).

Given the desired values and the feasible region R , the position of \underline{Y}^* , \underline{U}^* is determined by the weighting matrix Q .

Specifically, \underline{Y}^* , \underline{U}^* can be written as :

$$\begin{bmatrix} \underline{Y}^* \\ \underline{U}^* \end{bmatrix} = \begin{bmatrix} \underline{Y}^d \\ \underline{U}^d \end{bmatrix} - H N (N^T H N)^{-1} (N^T \begin{bmatrix} \underline{Y}^d \\ \underline{U}^d \end{bmatrix} - \underline{b}) \quad (2.8)$$

Let Ω denote the set of admissible values of $(\underline{Y}, \underline{U})$ from the policy maker's point of view and let $\Omega \cap R$ be nonempty and convex. Let Q_c denote the current weighting matrix in (2.2). The corresponding solution of (2.1), given by (2.8) with $Q=Q_c$, will be called "the current optimal solution".

$$(\underline{Y}_c^*, \underline{U}_c^*) \quad (2.9)$$

After a careful inspection of the values in (2.9), the policy maker may decide that some of these values are not quite what he wants them to be. This means that

$$(\underline{Y}_c^*, \underline{U}_c^*) \notin \Omega.$$

The policy makers' dissatisfaction, expressed in terms of qualitative information (i.e., inflation too high, interest rates too high from the point of view of "friendly" competitors, etc..), must now be translated into modifications of the weighting matrix. This for at least two reasons : firstly because each such expression of dissatisfaction, in general, implies an ordering or a reordering of priorities; secondly, given the feasible region R delineated a priori by the model equations, the only possibility for obtaining modified and more acceptable values for target variables would be to realign priorities!

Thus to obtain an optimal solution which the policy maker prefers to (2.9) is to alter some of the elements of Q_c . The basic idea is to update Q_c to a new weighting matrix Q_n such that "the new optimal trajectory"

$$(\underline{y}_n^*, \underline{u}_n^*), \quad (2.10)$$

obtained by solving (2.1) with $Q=Q_n$, is also in Ω . This is indicated in Figure 1 below. The simple conjecture that the new optimal solution may also not be completely satisfactory (i.e. not in Ω) as a result of the first update of Q leads to an iterative updating procedure. After each update, the optimisation problem is solved using the new Q until a solution is obtained which is also in Ω . The process of adjusting the weights may be repeated until a satisfactory analysis is made in order to arrive at such a solution.

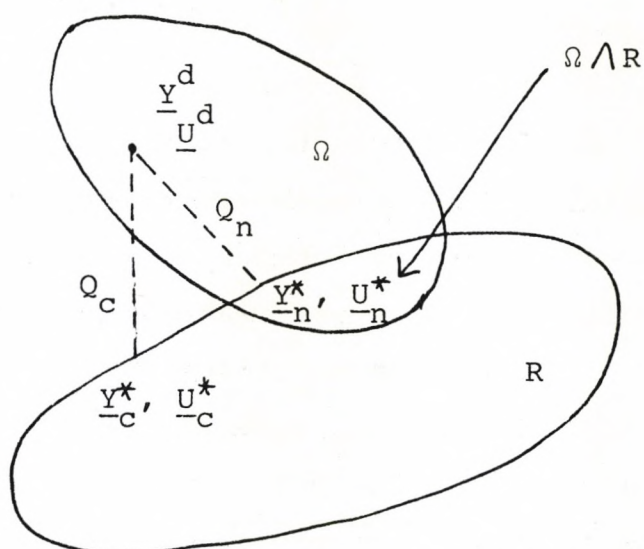


FIGURE 1

c) On the Specification of the Policy Makers' Preferences:

Computationally the iterative procedure can be summarised as follows:

Step 0 : Given the derived trajectory $\underline{y}^d, \underline{u}^d$ assume some initial weighting Q_c .

Step 1 : Using Q_c solve the minimisation problem (2.1) to obtain $\underline{y}_c^*, \underline{u}_c^*$.

Step 2 : Show $\underline{y}_c^*, \underline{u}_c^*$ to the policy maker. If $\underline{y}_c^*, \underline{u}_c^* \in \Omega$ then the problem is solved: stop. Otherwise, ask the policy maker to specify the trajectory he prefers to $\underline{y}_c^*, \underline{u}_c^*$. This "preferred" trajectory, denoted by

$$(\underline{y}^p, \underline{u}^p) \quad (2.11)$$

incorporates all the corrections to $\underline{y}_c^*, \underline{u}_c^*$ such that (2.11) is preferable to the "current" optimal trajectory. An obvious property of (2.11) is that $(\underline{y}^p, \underline{u}^p) \in \Omega$.

Step 3 : Use the "displacement" vector

$$\underline{\delta} \triangleq \begin{bmatrix} \underline{y}^p \\ \underline{u}^p \end{bmatrix} - \begin{bmatrix} \underline{y}_c^* \\ \underline{u}_c^* \end{bmatrix}$$

to update Q_c and obtain

$$Q_u = Q_c + \mu \frac{Q_c \underline{\delta} \underline{\delta}^T Q_c}{\underline{\delta}^T Q_c \underline{\delta}}$$

where $\mu \geq 0$ is a scalar chosen to emphasize the update term.

Set $Q_c = Q_u$ and go to Step 1.

Figure 2 below illustrates the iterative nature of the method. If the new optimal trajectory is not quite in Ω , the procedure of adjusting Q has to be repeated.

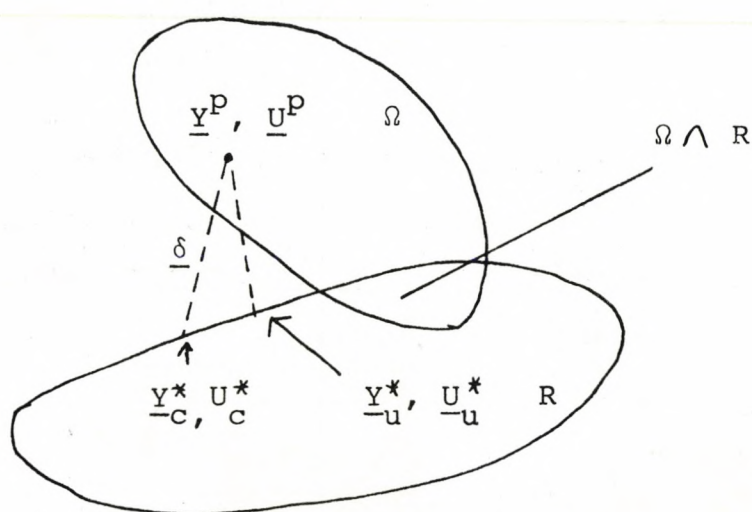


FIGURE 2

The fact that updating Q_c does indeed push y_u^*, u_u^* towards Ω , as depicted in Figure 2, is established in Rustem, Velupillai and Westcott (1978) along with numerical results. The extension of the method to general convex constraints is discussed in Rustem and Velupillai (1983).

It is clear then that given only preferred directions for key target variables and an element of consistency on the part of the policy maker (cf., for example, Sen (1970), p. 63), it is possible to translate this qualitative information into quantitative modification of the weights. The end result is a 'truer' appreciation of the interlocking nature of weights,

targets and instruments and indeed an awareness of the quantitative magnitude of their interdependence.

§ 3. Concluding Notes .

Researchers using optimization techniques in policy studies seem not to have been sufficiently careful and critical in the specifications of the objective function for a policy maker. Achievement of the targets of macroeconomic policy depend crucially on the priorities attached to them. If these priorities are misspecified, policy is suboptimal in a much more fundamental sense than the usual reasonings underlying 'second-best' arguments. 'Second-best' arguments refer to the region made feasible or not by the model equations, i.e., in our notation R . However, more basic especially from a political economic point of view is the region defined by Ω .

Starting from the problem posed by Markin and taking the hints provided by Frisch and Tinbergen we have attempted to tackle the problem of correctly specifying the region defined by Ω (i.e., the weights in a criterion function). This was done by taking into account explicitly the interdependence between targets, instruments and weights by using the implicit information given by a possible and implementable dialogue between a policy maker and a model builder. In this we have tried to remain as close as possible to the programme plea made by Frisch in his Nobel lecture.

At a less general and more technical level it must be pointed out that the usual criticisms levelled against the use of a

quadratic criterion function, in our context, lose much of its force (cf. Makin (op. cit p. 728, f.n. 18). The reason is simple. The quadratic criterion function is said to penalise deviation of the optimal solution from the desired direction as much as a deviation in an undesirable direction. Thus, for example, a deviation of the unemployment target from the desired level is penalised equally in either direction because of the symmetric nature of the quadratic criterion function. However, the iterative nature of our method shows clearly that such cases are the result of a misspecification of the desired values. If an optimal trajectory is equal to or higher (in the desirable direction) than its corresponding desired value specified in generating the original solution, then at the next iteration a new optimisation criterion may be defined by resetting those desired values marginally higher than the current optimal values. Such respecification eventually ensures that all departures from the desired values of the key target variables are in undesirable directions. This particular respecification is an element of the process of learning about the nature of the interdependence represented by the region R. Thus, the problems introduced by the symmetric nature of the quadratic criterion function should not be considered crucial within the context of an iterative scheme such as is represented above.

Recognizing that priorities attached to specific target variables in economic policy are crucial in determining the feasibility or not of policy instruments we have, in this paper, attempted to provide a solution to one aspect of the problem : viz, obtaining information on the correct specification

of these priorities. There remains, however, the more fundamental problem of Arrow impossibilities and the whole heavy weight of the criticisms launched by the New Classical Macroeconomists against any sort of discretion in formulating economic policy.

FOOTNOTES

- (1) cf. Kornai (1975), p. 420 for a related description of the problem in the context of planned economies.
- (2) There is, of course, an analogy with the Walrasian tatonnement. cf. Goodwin (1952), pp. 59-60 and Patinkin (1965), p. 39, f.n. 8 and p. 532, f.n. 2.
- (3) Technically, as will be seen below, the corresponding corrections are rank-one corrections.
- (4) Nonsymmetric matrices are equally acceptable since in this case the optimization problem can be equivalently formulated using $Q = \frac{1}{2}(Q_{NS} + Q_{NS}^T)$ where Q_{NS} is the nonsymmetric matrix.
- (5) The positive definiteness assumption on Q can be relaxed by considering instead of $(\underline{Y}, \underline{U})$ a set of variables in reduced dimension, transformed using the constraints. In effect this implies that Q needs to be positive definite only in the intersection of the constraints defining R . This overcomes difficulties of having to include all elements of $(\underline{Y}, \underline{U})$ in the objective function.
- (6) If some of the column vectors are linearly dependent on the others then there exist numerical procedures which detect this and consider only a linearly independent subset (see Gill and Murray, 1978; Rustem, 1981).
- (7) Dynamic problems can also be formulated in terms of (2.1) by setting :

$$\underline{y} \triangleq \left[\underline{y}^T(1), \dots, \underline{y}^T(K) \right]^T ; \underline{u} \triangleq \left[\underline{u}^T(1), \dots, \underline{u}^T(k) \right]^T$$

for a k-period discrete-time optimization problem where $\underline{y}(i)$, $\underline{u}(i)$ are the endogenous variables and policy instruments at time period i . The model equations (2.3), e.g. given by a standard econometric model, describing the region R can also be similarly written in stacked form.

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